DOCUMENT RESUME

ED 089 988 SE 017 520

TITLE	Mathematics Grades K-3: A Teacher's Guide.
INSTITUTION	Buffalo Public Schools, N. Y. Div. of Curriculum Evaluation and Development.
PUB DATE	68
NOTE	43p.
EDRS PRICE	MF-$0.75 HC-$1.85 PLUS POSTAGE
DESCRIPTORS	*Course Content; Curriculum; *Curriculum Guides; *Elementary School Mathematics; *Instruction; *Instructional Materials; Teaching Guides

ABSTRACT

This curriculum guide presents the outlines for course content in mathematics in grades K-3. For each grade level general overviews are given of the goals and objectives of the course. A detailed explanation of the content outline includes suggestions as to method of presentation. The mathematical concepts are explained using a technically correct approach. This is intended primarily for the teacher, so that the foundations she builds with the pupils in an informal way are based on sound, accurate mathematical principles. (JP)

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FOREWORD

The Buffalo Public Schools have had a very successful mathematics program for many years. Our students have been achieving at a rate comparable to other public schools throughout the state and our teachers are to be complimented for maintaining this fine record.

New developments in the field of mathematics, however, demanded that teachers and administrators review the existing program and make necessary changes and adaptations.

A Curriculum Committee under the direction of Mr. Louis Scholl, Director of Mathematics, has done an excellent job in preparing this new tentative Elementary Mathematics Guide. I feel the Committee has made a successful transition to modern mathematics with its increased emphasis on reasoning and analysis and at the same time continuing emphasis on skills and understandings needed in our contemporary society.

This Guide is tentative. Comments and suggestions will be welcomed from teachers and parents throughout the school year. The Curriculum Committee will review these suggestions and make changes that are deemed necessary.

I ask all elementary teachers to work diligently in helping children achieve success with this new program.

Joseph Manch
Superintendent of Schools

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OVERVIEW

LONG BEFORE children come to school they are acquainted with form and numbers. Their first experiences in life concern the shape of things around them. Very soon they are concerned with the size and number of objects. Almost all children love number mathematics before any formal teaching begins. You will find that an excellent opportunity to begin developing the child's power of reason lies in the area of mathematics. To take this intuitive feeling of the child, to cherish it while helping it to grow, is an awesome challenge to the kindergarten (or pre-school) teacher.

Modern mathematics contains a new vocabulary. To master this vocabulary and to understand the related concepts is an important step for the kindergarten teacher. Using correct mathematical words in an easy and natural way in many classroom activities, will help the children to understand the terms and clarify meanings so as to build a foundation for later more rigorous mathematics. The program in kindergarten is flexible and informal. It is important that the teacher at this stage use a correct mathematical vocabulary so that the children may associate proper mathematical terms with the concrete experiences which they feel and perceive in regard to form and number.

AIMS:

1. To develop a knowledge of number by using sets of objects.
2. To develop a formal awareness of number and the use of related numerals.
3. To build readiness for addition and substraction.
4. To discover distinguishing properties of common geometric forms.
5. To build readiness for measurement.
6. To build readiness for the concepts of "equal", "is greater than," "is less than."
7. To build readiness for mathematical reasoning through the solution of simple problems.

MATHEMATICS - KINDERGARTEN

Vocabulary

A. Distinguish informally between number and numeral

B. Introduce in an informal way:

above	between	equal to	none
below	before	greater	nothing
top	next	many	old
middle	begin	taller	older
bottom	each	up	same as
bigger	enough	down	match
smaller	lower	little	sentence
longer	higher	least	corner
as long as	more than	left	young
shorter	less than	right	younger

center	on	part
circle	in	pattern
cube (block)	path	set of points
empty	point	shape
empty set	rectangle	sphere (ball)
equal set	round	square
inside	set	straight
outside	member	triangle
out	side	zero

coin	size	time	today
cent	length	clock	yesterday
penny	measure	o'clock	tomorrow
nickel	ruler	long hand	
dime		short hand	
		hour	

cardinal number names -
zero through ten

ordinal number names -
first through fifth

I. Sets and Set Notation

A. Recognize sets of objects described.

B. Compare sets using one-to-one correspondence.

C. Introduce number as the common property of equivalent sets.

D. Arrange sets in order of size to establish sequence of counting numbers.

E. Introduce concept of the empty set to establish the idea of zero as a number.

II. Numeration System

A. Introduce informally Hindu-Arabic numerals 0 through 9 (set of digits)

B. Sequence of whole numbers through five

III. Operations on Sets

A. Joining sets (union)

B. Separating sets

C. Recognizing subsets, in sets of up to five members, as readiness for addition and subtraction.

IV. Geometry and Measurement

A. Nonmetric Geometry

1. Develop informally and intutively

a. A point as an exact location and a path line as a set of points.

b. A closed path such as a circle or rectangle locating points inside, on, and outside.

2. Recognition of shapes -

a. In a plane: square, circle, rectangle, triangle

b. In space: cube, sphere

B. Preparation for measurement (Metric Geometry)

1. Compare size of objects

2. Introduce concept of measurement

3. Telling time to the hour

V. Problem Solving

A. Provide opportunities for recognizing the usefulness of mathematics in daily life.

B. Informal introduction of mathematical problems in story form.

INSTRUCTIONAL OUTLINE

The five sections which follow are detailed explanations of the preceding five sections of Course Content, and include suggestions as to method of presentation.

While this instructional outline for kindergarten contains many technical terms and ideas, it cannot be emphasized too strongly that this work, as far as the children are concerned, is taken up in an incidental and informal way. The basic purpose of mathematical instruction in kindergarten is readiness.

This outline explains the mathematical concepts using a technically correct approach. This is intended primarily for the teacher, so that the foundations she builds with the pupils in an informal way are based on sound and accurate mathematical principles.

1. Sets and Set Notation

A set is simply a collection of objects. Those things which are in a set are called members or elements of the set. We have a set of pupils, a set of dishes, a set of chairs. The boys in the set of pupils form a subset of the set of pupils. The cups in the set of dishes form a subset of the complete set of dishes.

By pairing (matching) the members of two sets it is possible to discover without counting, whether one set has more than, fewer than, or as many members as the other. When the members of two sets can be paired exactly one-to-one (i.e. each set has exactly as many members as the other), the sets are called equivalent sets.

Sets that have exactly the same members are called equal sets, that is, they are what we would consider identical. Equivalent sets have the same number of members but not necessarily the same members, as for instance a set of five boys and a set of five chairs.

Non-equivalent sets can be arranged in order when the comparisons "less than" and "more than" are recognized. This leads to the ordering of numbers and counting. The number of elements in each set is called the cardinal number of the set. The empty set contains no members and has zero as its cardinal number.

We carefully distinguish between the set itself, and the number of members in the set. When we wish to refer to the cardinal number of a set, rather than the set itself, we write the letter N before the braces. For instance, N { a,b,c } means the number 3, and N { } means the number zero.

One description of the empty set might be the "set of elephants here in our classroom." Another example would be an empty box containing no objects, as compared to a box containing some objects.

All these ideas should be taken up informally. For instance, the children should not be expected to distinguish between equal and equivalent sets, or even to use these terms. However, where the teacher has a correct knowledge of these important mathematical ideas, she can more effectively build number readiness in her guidance of the children.

II. Numeration System

Counting is informal, incidental and part of planned and unplanned conversation. As children see the difference between an object and its name, they will begin to learn to distinguish between the number (an abstract idea) and the numeral (which is the symbol for the number) just as they have in the case of the boy Jim and his name Jim. Each digit is assigned as a number name to describe any set which has exactly that number of members, that is, it is the cardinal number of the set. For instance, a set of five rabbits, or five desks, or five chairs all have the number five assigned to them. Recognize and develop the idea that counting involves the concept of "adding one more."

III. Operations on Sets

In joining and separating sets of concrete objects we are preparing for addition and subtraction. The child can count to find the number of members that resulted from their actions.

IV. Geometry and Measurement

Working with concrete objects, children recognize circles, squares, triangles and rectangles in a plane by line drawing and paper patterns. They recognize cubes and spheres in space by handling blocks and balls.

By noting their place with reference to a closed path such as a circle or a rectangle (perhaps drawn on paper or part of a tile floor) the child can begin to grasp the meanings of such concepts and related vocabulary as point, line, inside, outside, and on.

When discussing such geometric concepts as point or line, the teacher must be careful not to create the impression that these are physical or material things. The markings ——— or • are only ink representations of the mathematical ideas of line and point. A point has no dimensions and no thickness, whereas the representation of a point does. A line has only one dimension - length, whereas the representation of a line must have some width if it is to be perceived. The situation is similar to the distinction between number and numeral. Statements such as "Let's draw a picture of a circle" will help to create the proper impression.

Many opportunities both planned and unplanned will arise which will enable the child to make comparisons, prepare for measurement, and become aware of the clock and measuring time. By being alert for such situations the teacher can seize opportunities for guidance and discovery.

V. Problem Solving

Everyday problem situations involving mathematical concepts frequently occur in the course of ordinary classroom activities. The teacher should take advantage of this by involving the children in the solution of the problems. Examples of this are checking the number of pupils present each day, or how many chairs are needed for a group of pupils, or perhaps even planning a party.

Number and size are important in the world around us. The teacher should seize every opportunity to do informal problem solving in mathematics with the children.

OVERVIEW

Many children come to grade one with some understanding of mathematics. Others come with only a vague understanding and some come with almost no understanding at all. Thus, the first grade teacher faces a dual task. First she must carefully and thoroughly review concepts and vocabulary contained in the kindergarten outline, using the correct phraseology at all times. Next she must continue to develop mathematical concepts through the use of concrete examples and physical experiences. The child must be guided to discover for himself the abstract meaning of a concept.

By mastering the new vocabulary and symbols of modern mathematics, the teacher will be better able to gain insight into the concepts to be taught. When sharing this knowledge with the class, the language used must always be appropriate and correct. Through the teacher's example the students will begin to build vocabulary and language patterns into their own thinking. Then written symbolism can be related to oral experiences.

AIMS:

1. To develop a knowledge of sets.
2. To develop concepts of cardinal and ordinal numbers.
3. To develop an understanding of place value.
4. To learn how to add and subtract whole numbers.
5. To build readiness for multiplication and division.
6. To discover properties of our number system.
7. To establish various geometric properties and develop skill in measurement.
8. To develop the use of open sentences.
9. To build readiness for estimating and mental arithmetic.
10. To build readiness for analytical reasoning through the solution of simple problems.

MATHEMATICS - GRADE ONE

Review Mathematical Concepts Covered in Kindergarten

Vocabulary and Symbolization

A. Review and reinforce vocabulary of previous year.

B. Introduce terms:

sum	one-half	penny
difference	one-third	cent
add	one-fourth	nickel
subtract	hour	dime
set	half-hour	cup
number of members	half-past	pint
more	calendar	quart
less	year	inch
before	month	foot
after	week	dozen
inside	day	
outside		

C. Introduce terms and symbols:

set braces	point	geometric figures
union of sets	end-point	quadrilateral
difference of sets	line	pentagon
equal sets	line segment	circle
empty sets	number line	triangle
equals	arrowhead (to show that a line goes on and on)	rectangle
plus		square
minus		
dots (to represent points)		

I. Sets and Set Notation

A. Designate sets with braces.

B. Establish one-to-one correspondence between equivalent sets.

C. Develop abstract concept of number. Develop the idea that each set is associated with a cardinal number.

D. Study sets with a single member - (a set or group may consist of one member)

E. Determine cardinal number of a set.

F. To develop concepts:

1. Inequality - compare sets with different numbers of members. (greater than and less than)

2. Zero - study empty set (use of braces with no member)

G. Associate sets of tens and ones with corresponding two-digit numerals.

H. Union of sets.

II. Numeration System

A. Names of the whole numbers through 100

B. Learn the following sequences, and use these to develop the concepts of before, after, and between:

1. To 100 by 1's

2. To 20 by 2's

3. To 100 by 10's

4. To 50 by 5's

5. To 15 by 3's

6. To 20 by 4's

C. Counting backwards from 10 to 0

D. Introduce concept of odd and even numbers

E. Introduce abacus

F. Develop ordinals first through sixth

G. Develop concept of ten as the unit of grouping in the decimal numeration system by counting sets of ten (place value).

H. Interpret two-digit numerals in terms of tens and ones.

I. Discover that numbers represented by several digits depend upon each digit's place in the numeral.

J. Use hundred chart to relate a two-digit numeral to the number of tens and ones it represents.

K. Write one-digit and two-digit numerals in sequence along number line.

III. Operations

A. Whole Numbers

1. Addition and Subtraction

a. Discovering and mastering addition and subtraction facts through sums of 10, building readiness for sums up through 18.

1) to introduce the property of zero (identity element) for addition and subtraction.
2) joining and separating sets
3) using labeled number line
4) reading and writing number sentences

b. Introduce horizontal and vertical ways of writing addition and subtraction.

c. Addition is commutative; subtraction is non-commutative.

d. Develop addition with three addends.

1) use of associative law

e. Addition of tens

1) by counting by tens
2) by counting backwards by tens from 100 to 0

f. Develop subtraction as the inverse of addition, by:

1) separating sets
2) using labeled number line
3) reading and writing number sentences

g. Use number sentences to teach related facts

B. Fractional Numbers

1. Introduce the fractions one-half, one-third and one-fourth.

2. Use the geometric approach, by dividing circle shapes, square shapes, and rectangle shapes and others into halves, thirds, and fourths.

3. Use sets of objects to separate into equivalent subsets, showing halves, thirds, and fourths.

4. Use fraction notation with horizontal line.

5. Reinforce meaning of one-half by measuring time to half-hour.

IV. Geometry and Measurement

A. Metric Geometry

1. Develop appreciation for need of measurement and units of measure.

2. Cover the following:

a. Telling time to hour and half-hour.

b. Counting five-minute intervals on a clock.

c. Reading and using calendar.

d. Using an inch and foot as a unit of length.

e. Using a ruler to measure lengths of line in feet and inches.

f. Comparing volumes using cups, glasses, pints, quarts.

g. Teach value of penny, nickel, dime, cent.

h. Develop concept of dozen.

B. Non-Metric Geometry

1. Develop ability to recognize rectangles, squares, triangles, and circles, cubes and spheres, through identification of their properties.

a. A plane figure consists of the line segments or curves which bound a region. These line segments and curves are themselves defined as sets of points following a particular path.

b. Establish idea that the region within a plane figure, such as triangle or circle is not part of the plane figure.

2. Present the following ideas:

a. A drawn line is a picture or model of a theoretical line which has length but not width.

b. A point is a place in space.

c. A line is a set of many points.

d. Only one straight line can be drawn through two points.

e. A line segment has a fixed length between two end-points.

V. Number Sentences

A. Equations

1. To emphasize inverse relationship between addition and subtraction.

2. To develop commutative property of addition.

3. To use in problem solving.

B. Solution of Open Sentences

VI. Estimation and Mental Arithmetic

A. Build readiness in everyday classroom activities.

B. Recognize sequential patterns.

VII. Problem Solving

A. To verbalize everyday classroom activities involving number.

1. Use number line.

2. Use number sentences.

B. Interpret and solve verbal problems informally

Instructional Outline

(The seven sections which follow are detailed explanations of the preceding seven sections of Course Content, and include suggestions as to method of presentation).

I. Sets and Set Notation

A set is simply a collection of objects or ideas. In the set $\{a, b, c\}$ the letter c is a member or element of the set. One subset of this set is $\{a, c\}$. It uses only members from the original set.

By pairing (matching) the members of two sets it is possible to discover without counting, whether one set has more, fewer, or as many members as the other set. When the members of two sets can be paired exactly one-to-one, the sets are in one-to-one correspondence. Such sets are called equivalent sets.

Sets that have exactly the same members are called equal sets. Equivalent sets have the same number of members but not necessarily the same members.

Non-equivalent sets can be arranged in order when the comparisons less than or more than are recognized. This leads to the ordering of numbers and counting. The number of elements in each set is called the cardinal number of the set. A set may have more than one member, exactly one member (cardinal number one), or no members (the empty set - cardinal number zero). A description of this last set might be "the set of boys over eight feet tall in our classroom."

We carefully distinguish between the set itself, and the number of members in the set. When we wish to refer to the cardinal number of a set, rather than the set itself, we write the letter N before the braces. For instance $N\{a, b, c\}$ means the number 3, and $N\{\ \}$ means the number zero.

II. Numeration System

We begin developing the abstract idea of a number through the use of concrete materials which we arrange in sets. We may arrange several equivalent sets each having four members to develop the abstract idea of 4. In this way we develop the set of natural (counting) numbers, $\{1, 2, 3....\}$. We establish zero as the number of the empty (null) set, and put this in with the set of natural numbers. We then have the set of whole numbers, $\{0, 1, 2, 3....\}$. These sets can be developed visually on a number line.

Care should be taken so that students do not think of the points on the line as being the numbers. Accurate wording to the effect that we are matching points and numbers to make a picture should be used.

Of fundamental importance to all future work and success is the development of the number 10 as the base of our decimal number system. A great amount of time and energy must be spent developing through concrete experiences the concept of grouping by tens.

III. Operations

In joining and separating sets of concrete objects the children are preparing for addition and subtraction. The inverse relationship between these operations (they are the opposite of one another) should be discovered by the students. Careful guidance will also allow the children to discover (without introducing formal vocabulary) the commutative property of addition, the associative property of addition and the identity element for addition.

Commutative Property: The order of the addends does not change the sum. For instance $4 + 3 = 7$ and $3 + 4 = 7$.

Associative Property: The manner in which we group three addends does not change the sum. For instance, in adding $5 + 2 + 6$, we may first add 5 and 2 to get 7, then add the 7 and the 6 to get the final sum 13. Or else we may first add 2 and 6 to get 8, and then add the 8 and 5 to get 13, that is

$$(5 + 2) + 6 = 5 + (2 + 6)$$
$$7 + 6 = 5 + 8$$
$$13 = 13$$

This should be done informally with the children at this grade level. This topic is ideal for the guided discovery technique in teaching mathematics.

Identity Element: The sum of any number and zero is equal to the original number. e.g. $5 + 0 = 5$. Hence zero is the identity element under the operation of addition.

IV. Geometry and Measurement

Metric geometry refers to that type of geometry in which measurement is involved, such as the length of a line, the size of a room, or the equality of two rectangles. Nonmetric geometry has to do with the shapes only of geometric figures, that is, whether a figure is square or round, or whether it is a sphere or cylinder, and whether a line is straight or curved.

The work of comparison begun in kindergarten is extended as is the recognition of basic geometric forms.

When discussing such geometric concepts as point, line or triangle, the teacher must be careful not to create the impression that these are physical material things. The markings ——— or • are only ink representations of a mathematical idea. A point has no dimensions, (a dot or period does) and a line has only one dimension - length (a pencil or chalk "line" also has width). The situation is similar to the distinction between number (idea - immaterial) and numeral (symbol - material). For instance, 5 and V are the same number, but are different numerals. Statements such as "Let's draw a picture of a triangle" will help to create the proper impression.

In the representation of a triangle, only the set of points on the three line segments actually comprise the triangle. The region enclosed by the three line segments is not considered a part of the triangle. Hence when the children are instructed to "Color the triangle," it is understood this is to mean that the children are to color the three line segments which comprise the triangle, but are not to color inside the triangle.

V. Number Sentences

A variable or frame (such as _____, $\square$, m) is a symbol placed in a number sentence indicating that a number is to be inserted so as to make the number sentence true. A sentence which contains such a symbol is called an open sentence. Such open sentences are extremely valuable to provide understanding and meaningful practice of number relationships.

VI. Estimating and Mental Arithmetic

Oral questions about planning various activities, or questions of comparison will help build readiness for more involved mental activities. Problems involving sequences such as 2, 4, 6, __?__ are valuable in developing understanding of number relationships as well as serving as an introduction to mathematical reasoning.

VII. Problem Solving

Simple problems such as "If John brings 2 goldfish and June brings 3 goldfish how many will we have?" will give the students the opportunity to write such number sentences as $2 + 3 =$ __?__. Such exercises develop many skills including the skill of organizing a set of facts.

OVERVIEW

The children who come to you in second grade are ready to continue their study of mathematics. They have been introduced to the fundamentals of formal arithmetic and geometry and hopefully have made some important discoveries in each area and are eager to proceed. While taking care not to destroy this impatience to continue the story of mathematics, the teacher must nevertheless re-establish the concepts developed in previous years. A thorough understanding of basic concepts is developed with patience and skill, and often may seem slow in coming, but when it is finally achieved the reward is a more rapid and complete mastering of later work.

A modern course in mathematics contains new ideas and a special language for precise statements of these ideas. Both must be mastered by the teacher so that she may share this knowledge with her pupils. When ideas are stated properly using correct terminology not only is the correct thought implanted but misconceptions and misunderstandings are avoided.

AIMS:

1. To expand the use of set ideas as an aid in the understanding of number concepts and of geometry.
2. To reinforce and extend basic concepts of our numeration system.
3. To learn basic addition and subtraction facts.
4. To build readiness for multiplication and division.
5. To extend understanding of fractions.
6. To extend knowledge of geometric concepts and develop skill in measurement.
7. To expand the use of number sentences as an aid to the understanding of operations on numbers.
8. To develop the ability to make mental comparisons and estimate answers.
9. To extend skill in analyzing a given set of facts and arrive at conclusions from an organization of those facts.

MATHEMATICS - GRADE TWO

Review mathematical concepts covered in Kindergarten and Grade One.

Vocabulary and Symbolization

A. Reinforce vocabulary of previous years.

B. Mathematical terms and symbols:

number	plus	ruler	whole	line
numeral	minus	inch	one-half	line segment
number line	addition	foot	one-fourth	end point
odd number	subtraction	dozen	one-third	path
even number	equals	measure	o'clock	shape
set	times	cup	sum	square
subset	commutative	pint	difference	circle
union of sets	associative	quart	divide	triangle
member	less than	thermometer		rectangle
empty set	more than	temperature		pentagon
one-to-one correspondence	cent	degree		quadrilateral
	penny	calendar		convex
pairing	nickel	days of the week		concave
between	dime			perimeter
before	quarter	months of the year		area
after	half-dollar			
inside	dollar			
outside				

1. Sets and Set Notation

A. Designate sets with braces, and the cardinal number of the set with the letter N before the braces.

B. Develop the concept of equal sets, and equivalent sets.

C. Develop concept of inequality by comparing sets with different numbers of members.

D. Associate the number zero with the empty set.

E. Develop the concept of subset, using the correct symbol.

F. Associate sets of tens and ones with corresponding two digit numerals

G. Associate sets of hundreds, tens and ones with corresponding three digit numerals.

H. Study sets of odd and even numbers.

II. Numeration Systems

A. Counting

1. Learn in order the names of whole numbers through 200.

2. Count by 1's, 2's. 3's. 4's, 5's, and 10's to 200.

3. Read and write in order, numerals to 200.

4. Group recognition through 10.

5. Reteach the 10's in 20, 30, etc. to 100.

6. Read and write numerals from 101 to 200.

7. Understand place value to include three digit numbers in terms of hundreds, tens and ones, using expanded notation.

8. Understand ordinals first through twelfth.

B. Properties

1. Intuitive development of the commutative and associative laws.

2. Increase emphasis on the concepts of equality and inequality. Develop the concept that when two real numbers are compared, one number is either equal to, less than, or greater than, the other number.

III. Binary Operations

A. Whole Numbers

1. Addition and subtraction

a. Discover and use addition and subtraction facts, through sums up through 19.

1) join sets (union), and subtract sets where one is a subset of the other
2) use of the number line in addition and subtraction
3) read and write number sentences
4) introduce horizontal and vertical ways of writing addition and subtraction exercises

b. Introduce higher decade addition and subtraction using:

1) a counting chart
2) relationships involving the basic number facts
3) patterns - find the sum or difference between two numbers named by two digit numerals. (First develop using expanded notation)

c. Check subtraction by addition

2. Multiplication

a. Develop multiplication through use of repeated addends - join equivalent sets.

b. Use of the number line for multiplication.

c. Extend multiplication facts with application of the commutative property of multiplication.

B. Fractions

1. Extend understanding of the fraction.

2. Find $\frac{1}{2}$ of each even number from 2 to 18.

3. Introduce $\frac{1}{3}$ and $\frac{1}{4}$ (Use horizontal line)

4. Reinforce the meaning of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ by measuring line segments.

IV. Geometry and Measurement

A. Metric Geometry and Units of Measure.

1. Continue emphasis on selecting units of measure in terms of what is being measured.

2. Extend understanding of cent, nickel, and dime.

3. Introduce quarter, half dollar, and dollar.

4. Extend telling time by the hour, half hour, and in five minute intervals.

5. Relate 12 inches and one foot.

6. Introduce measure to the nearest inch.

7. Compare volumes using cup, half pint, pint, and quart.

8. Teach dozen and half-dozen.

9. Read a thermometer and develop the meaning of freezing, and below zero. Introduce degree as a unit of temperature.

10. Extend the study of the calendar to include days, weeks, months, and year.

11. Introduce intuitively and pictorially the concepts of perimeter and area.

B. Non-Metric Geometry

1. Extend concept of a point as an exact location and a line as a set of points.

2. Extend concept of a line segment as a set of points consisting of the two end points, and all the points on the line segment between these two end points.

3. Extend emphasis on properties of triangles, rectangles and squares.

4. Develop intuitively the concept of the circle as the set of points in a plane equally distant from the center.

V. Mathematical Sentences

A. Equations

1. Find solutions for given mathematical sentences.

2. Continue to emphasize the inverse relationship between addition and subtraction.

3. Extend commutative and associative property of addition.

4. Extend use of mathematical sentences in problem solving.

B. Inequalities

1. Continue the development of the concepts of equal to, and less than, and the symbols for these terms.

2. The symbol for is greater than is not introduced until the third grade.

VI. Estimating and Mental Arithmetic

A. Develop estimating and the ability to do mental arithmetic with application to real experiences.

B. Recognize sequential patterns.

VII. Problem Solving

A. Extend association of a numerical statement with pictures and drawings, and with written and oral statements of a problem.

B. Extend development of the ability to analyze problems and record facts to associate the correct operation with the problem situation.

C. Interpret problems.

D. Informal development in estimating answers and deriving answers through reasoning.

Instructional Outline

(The seven sections which follow are detailed explanations of the preceding seven sections of Course Content, and include suggestions as to methods of teaching.)

I. Sets and Set Notation

A set is simply a collection of objects or ideas. In the set $\{1,5,9\}$ the number 5 is said to be a member or element of the set. Examples of subsets of this set are $\{1,9\}$ and $\{5\}$. They contain only members found in the original set. We express this symbolically as $\{1,9\} \subset \{1,5,9\}$. We read this as follows: "The set $\{1,9\}$ is a subset of the set $\{1,5,9\}$."

By pairing (matching) the members of two sets, it is possible to discover without counting, whether one set has more than, fewer than or as many members as the other. When the members of two sets can be paired exactly one-to-one, (i.e. they have the same number of elements) the sets are in one-to-one correspondence. Such sets are called equivalent sets.

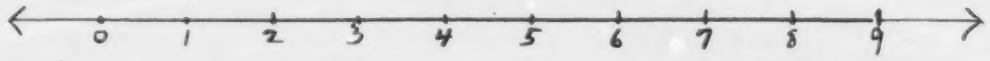
Sets that have exactly the same members are called equal sets. Equivalent sets have the same number of members, but not the same members.

Non-equivalent sets can be arranged in order when comparisons fewer than or more than are recognized. This leads to the ordering of the whole numbers. The number of elements in each set is called the cardinal number of the set. A set may have more than one member, exactly one member, or no members. This last set is called the empty (null) set and it has the cardinal number zero. A description of this set might be "the set of girls in our class who have visited Pakistan." Another example might be an empty container with no objects in it, as opposed to a container having one or more objects in it.

We carefully distinguish between the set itself, and the number of members in the set. When we wish to refer to the cardinal number of a set, rather than the set itself, we write the letter N before the braces. For instance $N\{a,b,c\}$ means the number 3, and $N\{\}$ means the number zero. This method of representing a cardinal number is an intermediate notation to the use of Arabic numerals.

II. Numeration System

A principal goal for second grade is the development of the concepts of order and place value. Counting is done to establish sequences mentioned in the outline. The number line will enable the students to visualize the set of whole numbers W = {0,1,2,3...}



Care should be taken so that the students do not think of the points as actually being numbers. Accurate wording should be used so that they realize we are matching sets (a set of points with a set of numbers) to make a picture.

In developing the meaning of place value, expanded notation should be used for instance 567 = 5 x 100 + 6 x 10 + 7 or 567 = 5 hundreds + 6 tens + 7 ones.

Through guided discovery, the children learn that the order of addends does not affect the sum, and that in multiplication, the order of the factors does not affect the product. Thus the children themselves discover the commutative property of addition and multiplication. This should be done by giving such examples as 2 + 5 = 5 + 2 and 2 x 3 = 3 x 2. This should also be shown on the number line, and using various visual aids. Likewise they discover that the way in which we group (associate) addends does not change the sum - the associative property of addition. Appropriate examples and the number line may be used to develop this concept.

III. Operations

In joining and separating sets of concrete objects the children are discovering the basic addition and subtraction facts while noting the inverse relationship which exists between these operations.

The development of multiplication proceeds from concrete experiences in much the same way. Structured sets (arrays), for instance 5 piles of books with 2 books in each pile, help the child to understand these operations. The notation 5 x 2 = 10 at this level refers primarily to repeated additions and not a complete mastery of the abstract concept of a new operation.

The work on unit fractions includes one-half, one-third and one-fourth. Concrete and visual aids are essential to a thorough foundation in the basic meaning of fractions.

IV. Geometry and Measurement

A variety of units of measure, simple conversions, use of measuring devices and various comparisons are part of the work to be developed in grade two. Recognition of basic geometric forms continues and the circle is introduced.

When discussing such geometric concepts as point, line or circle, the teacher must be careful not to create the impression that these are physical, material things. The markings _____ or • which you see on this paper are only ink representations of a mathematical idea. A point has no dimensions (a dot or period does) and a line has only one dimension - length (a pencil or chalk "line" also has a width). The situation is similar to the distinction between number (which is an idea and therefore immaterial) and numeral (which is a symbol and made of some material such as ink). Statements such as "let's draw a picture of a circle" will help to give the proper impression. It may be well to point out also that in a drawing of a circle only the set of points on the closed curve (path or line) constitute the circle. The region enclosed is not the circle nor any part of it.

V. Number Sentences

A variable or frame (such as _____ or □) is a symbol used to represent a definite but unspecified number (or numbers). A sentence which contains such a symbol is called an open sentence. This symbol for a variable may also be a letter such as m or x, or any other convenient letter. If the verb of the sentence is " = ", the sentence is called an equation and if the verb is "is more than" or "is less than", the sentence is called an inequality. For instance _____ + 4 = 10 is an equation and _____ + 3 < 7 is an inequality.

When the variable is replaced by a number from a prechosen set (replacement set) the resulting sentence is either true or false. Any number which makes the sentence true is a member of the solution set of that sentence, that is, it is an answer. For instance in the example above if the replacement set is $W = \{0,1,2...\}$ then for the equation, the solution set is $\{6\}$, and for the inequality $\{0,1,2,3\}$.

A solution set may contain no members, one member or more than one member.

VI. Estimating and Mental Arithmetic

Oral questions about planning various activities or questions of comparison will enable the child to develop powers of judgment and estimation. Problems such as 2,4,6, __?__ involving a variety of sequences will give the child the opportunity to develop several important skills.

VII. Problem Solving

When a problem such as "If John had 5¢ and spent 2¢ how much did he have left?" is given to the class they have the opportunity to develop skills analyzing and organizing facts. The latter is best done with a number sentence 5 - 2 = _____. The child should be encouraged to judge the reasonableness of his results.

OVERVIEW

The child entering third grade has been provided with a background of basic mathematical concepts patiently developed through two or three years of formal schooling together with the observations and experiences of his daily life. The groundwork already laid provides the teacher with the opportunity to broaden the child's interest and enthusiasm for mathematics. This can be done by presenting the story of mathematics in a coherent and logical form. A solid knowledge of grade three work will encourage the child to look forward to continuing this story next year.

The skillful teacher will seek to guide her pupils in discovering for themselves the fundamentals and detailed facts that are a part of the course content. Her goal will be to see that the students develop a keen awareness of the structure and logic of mathematics for she knows this will permit flexibility of application, promote retention of basic facts and instill a desire to continue its study. Her own intellectual curiosity will be contagious and encourage her pupils to investigate ideas and to seek answers.

AIMS:

1. To expand the use of set ideas as an aid in the understanding of number and geometry.
2. To reinforce and extend the basic concepts and properties of our base ten number system.
3. To expand addition and subtraction to include regrouping numbers.
4. To develop basic multiplication and division facts.
5. To extend and deepen the understanding of fractions.
6. To develop a broader familiarity with geometry.
7. To extend use of number sentences.
8. To develop skill in estimating and mental computation.
9. To strengthen the ability to analyze and organize facts presented in problems, and to arrive at correct conclusions.

MATHEMATICS - GRADE THREE

Review mathematical concepts covered in kindergarten, and grade one and two.

Vocabulary and Other Symbolization

A. Reinforce vocabulary of previous years.

B. Mathematical terms and symbols:

whole number	geometry	Roman numerals
numeral	geometric figure	quarter of (time)
digit	model	noon
place value	plane figure	midnight
expanded form	closed figure	yard
equation	convex	ounce
mathematical sentence	concave	one-fifth
addend	edge	one-sixth
regrouping	vertex	one-eighth
factor	inside	commutative
multiplying	outside	associative
product	on	distributive
multiple	perimeter	equation
division	area	is less than
estimate	degree	is greater than
fraction	braces	parentheses
union of sets		
difference of sets		
intersection of sets		

C. Abbreviations

A. M.	in.
P. M.	ft.
yd.	gal.
pt.	lb.
qt.	oz.

I. Sets and Set Notation

A. Review the use of braces to identify sets. The use of the letter N before the braces designates the cardinal number of the set.

B. Review the empty set which has the cardinal number zero.

C. Review equal sets and equivalent sets.

D. Review the concept of subset.

E. Review and extend inequalities.

F. Develop the concepts of union of sets, difference of sets, and intersection of sets.

G. Identify and work with:

1. The set of whole numbers.

2. A set of even numbers, and a set of odd numbers, as subsets of the whole numbers.

3. A set of multiples of a number.

4. A set of points in a plane or in space.

II. Numeration System

A. Counting

1. Maintain counting developed in earlier grades.

2. Review and extend the decimal place value system to include practice with four, five and six-digit numerals, and using expanded notation.

3. Count by tens and hundreds to one-thousand.

4. Use of the abacus.

5. Compare the Roman numeration system with the Hindu-Arabic system, pointing out that the Roman system did not have a zero, and hence could not have a complete place value system.

6. Extend numeration to six-digit numerals.

7. Develop the concept that a number may be named in many ways, using various combinations of numerals.

B. Properties

1. Review and extend applications of the commutative property of addition and multiplication, and introduce the associative property.

2. Review zero as the identity element in addition.

3. Introduce one as the identity element for multiplication and its related role in division.

4. Introduce the distributive property of multiplication with respect to addition.

5. Develop an understanding of the role of zero in multiplication.

III. Binary Operations

A. Whole Numbers

1. Addition and Subtraction

a. Continue development of addition by relating the addition of whole numbers in terms of the joining (union) of disjoint sets.

b. Continue the development of subtraction as the inverse operation of addition.

c. Extend the development of addition and subtraction to two and three digit numerals using expanded notation.

d. Regrouping ten ones as a ten and ten tens as a hundred in addition (carrying).

e. Regrouping a ten as ten ones, and a hundred as ten tens in subtraction (borrowing).

f. Continue the development of mental computation for finding answers in addition and subtraction.

g. Continue the practice of estimating answers.

2. Multiplication and Division

a. Present and master the basic multiplication facts from $0 \times 0 = 0$ through $9 \times 10 = 90$.

b. Let pupils discover the commutative property of multiplication.

c. Introduce the concept that the product of any two whole numbers is a multiple of each of those whole numbers.

d. Teach multiplying multiples of ten.

e. Teach the multiplying of a two digit number by a one digit number through use of the distributive property.

f. Develop the ability to estimate products.

g. Stress the inverse relation between multiplication and division.

h. Develop the writing of related multiplication and division sentences for each pair of factors.

i. Provide practice with related division facts using:

1) arrays
2) repeated subtraction
3) the number line

j. Introduce short division with no remainder.

k. Develop division with multiples of ten as the dividend.

l. Extend multiplication and division to include problems involving money.

B. Fractions

1. Develop understanding of fractions as expressing a part of a whole.

2. Extend unit fractions to include $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{8}$.

3. Develop meaning for fractions other than unit fractions with denominators 2,3,4,5,6, and 8.

4. Associate fractions with points on a number line.

5. Compare fractions (greater than or less than).

6. Introduce the concept of equivalent fractions through pictorial representation.

IV. Geometry and Measurement

A. Metric Geometry

1. Continue the development of telling time by:

a. Relating sixty minutes to one hour.

b. Teaching the reading of minutes past an hour and before an hour.

c. Teaching the telling of time in one minute intervals.

2. Extend the development of measuring distance by:

   a. Relating twelve inches and one foot.

   b. Relating three feet and one yard.

3. Continue the development of the concepts of perimeter and area.

4. Continue the development of measure of weight relating sixteen ounces and one pound.

5. Continue the development of measure of temperature by comparing degrees of temperature.

B. Non-Metric Geometry

1. Continue the development of point, line, line segment, closed path.

2. Develop the properties of the circle.

3. Develop the concepts of open and closed figures.

4. Develop the concept of points inside, outside, or on a figure.

V. Mathematical Sentences

A. Equations

1. Solution of equations is intuitive, without formal rules.

2. Translation of English sentences into mathematical sentences.

3. Solution and checking of these mathematical sentences.

B. Inequalities

1. Teach the symbols and meaning for "is less than" ($<$) and "is greater than" ($>$).

2. Practice in finding those whole numbers which make an inequality true, that is, find the solution set for an inequality.

VI. Estimating and Mental Arithmetic

A. Estimating solutions in all types of problems before computions.

B. Practice in mental arithmetic.

C. Estimating answers to judge reasonableness of computed answer.

VII. Problem Solving

A. Develop the ability to translate quantitative sentences into mathematical sentences using symbols.

B. Teach techniques necessary to analyze verbal problems.

C. Apply concepts to problems without numbers, in which the emphasis is on which operation is to be performed.

INSTRUCTIONAL OUTLINE

The seven sections which follow are detailed explanations of the preceding seven sections of course content, and include suggestions as to the method of teaching.

I. Sets and Set Notation

A set is simply a collection of objects or ideas. These objects or ideas are called members or elements of the set, and are enclosed in braces. For instance, the set of whole numbers less than ten is shown as $\{0,1,2,3,4,5,6,7,8,9\}$.

The number of members which a set contains is called the cardinal number of the set. This is indicated by the letter N before the braces. For example, the set $\{a,b,c,d\}$ has the cardinal number 4, (four members in the set), and this would be written $N\{a,b,c,d\} = 4$.

If the members of one set can be paired exactly in one-to-one correspondence with the members of another set, then these sets have the same cardinal number, and are called equivalent sets. An example of this would be matching a set of 30 children in a class with a set of 30 desks in the classroom. The set of children would be equivalent to the set of desks. We only speak of equal sets when the two sets are identical, that is, they are made up of exactly the same members. For instance, the set $\{a,b,c\}$ is equal to the $\{a,c,b\}$.

The set with the cardinal number zero is written $\{\ \}$, or sometimes the Greek symbol ϕ is used. An example of this set might be "the set of elephants living in our classroom." If we wish to select only certain members of a given set, for example, if we wish to select the set of even whole numbers less than ten from the set of all whole numbers less than ten, we form the subset $\{0,2,4,6,8\}$. To indicate a subset we use the symbol "$\subset$". Hence in this example we write:
$\{0,2,4,6,8\} \subset \{0,1,2,3,4,5,6,7,8,9\}$. We would read this "The set $\{0,2,4,6,8\}$ is a subset of (or is contained in) the set $\{0,1,2,3,4,5,6,7,8,9\}$. We might also form the subset of odd numbers less than ten $\{1,3,5,7,9\}$. In this instance the union of these two subsets, indicated by the symbol "$\cup$", would be our original set, that is,
$\{0,2,4,6,8\} \cup \{1,3,5,7,9\} = \{0,1,2,3,4,5,6,7,8,9\}$.
However, in finding the union of two sets, we must be careful not to count any of the members of the sets twice. For instance, the union of the set of odd whole numbers less than ten $\{1,3,5,7,9\}$, with the set of whole numbers less than ten that are exactly divisible by three $\{0,3,6,9\}$, would be the

set $\{0,1,3,5,6,7,9\}$, that is
$\{1,3,5,7,9\} \cup \{0,3,6,9\} = \{0,1,3,5,6,7,9\}$.
Note that in the above example the members 3 and 9 are only counted once, although they appear in both sets. It may be noted that while the cardinal number of the first subset is 5, and the cardinal number of the second subset is 4, the cardinal number of the union of the two subsets is only 7. This will always happen when the two subsets are not disjoint, that is, they have some members in common.

The <u>difference</u> of two sets is analogous to subtraction. For instance we find the difference of the following two sets - the set of whole numbers less than 10, and the set of odd whole numbers less than ten, as follows:
$\{0,1,2,3,4,5,6,7,8,9\} - \{1,3,5,7,9\} = \{0,2,4,6,8\}$.
Note that to find the difference of two sets, it is more convenient if one is a subset of the other.

The <u>intersection</u> of two sets, indicated by the symbol " $\cap$ ", is defined as the set of only those members which are found in <u>both</u> sets. For instance, the intersection of the set of whole numbers less than ten with the set of even whole numbers greater than five and less than thirteen would be the set $\{6,8\}$, that is $\{0,1,2,3,4,5,6,7,8,9\} \cap \{6,8,10,12\} = \{6,8\}$.
As another example, the intersection of the set of odd numbers less than ten with the set of even numbers less than ten would be the empty set, since they have no members in common, that is $\{1,3,5,7,9\} \cap \{0,2,4,6,8\} = \{\}$.

II. <u>Numeration Systems</u>

The work outlined in this section centers on the concepts of order and place value and on the properties of the real number system. Expanded notation allows the pupil to better understand the concept of place value and the variety of values that a digit might have. In the example $447 = 4 \times 100 + 4 \times 10 + 7$ the pupil is able to see that the digit "4" has two different values because of the place value feature of our number system. Emphasize the idea that each place value is ten times as much as the place value to its right. In contrast, the Romans used a modified additive system. The Roman numeral VII does not mean $5 \times 100 + 1 \times 10 + 1$, but derives its value from the addition process $5 + 1 + 1$. It is to be noted that the number zero together with a symbol for it is essential to a place value system, and the Romans did not have a zero in their number system. A discussion with the class of the merits of each system may prove interesting and beneficial.

In previous grades the pupils have discovered and intuitively developed the commutative law for addition and multiplication. This property is now studied more formally, and the associative and distributive properties are now introduced.

A binary operation is one in which a third member is obtained from a pair of numbers. Using many examples, the children are led to discover that in the binary operation of addition, the order in which we add the two numbers can be changed without changing the sum, that is, $2 + 3 = 3 + 2$. This is the commutative law. They also discover that the operation of multiplication is commutative. Using examples, they will also find out that the operations of subtraction and division are <u>not</u> commutative.

When we operate on three or more numbers, we use the associative law. For instance, if we are given three numbers to add, we may pair them up in two different ways. We indicate pairing with parentheses, as $2 + 5 + 4$ may be paired as $(2 + 5) + 4$ or as $2 + (5 + 4)$. Hence we have the two possibilities

$(2 + 5) + 4$	$2 + (5 + 4)$
$7 + 4$	$2 + 9$
11	11

In each case we obtain the same answer - eleven, and this example illustrates the fact that addition is associative. In the same way we can show that subtraction is <u>not</u> associative. For instance, if we have the example $12 - 5 - 3$, we may show this in two different ways:

$(12 - 5) - 3$	$12 - (5 - 3)$
$7 - 3$	$12 - 2$
4	10

Here we obtain two different answers by pairing the numbers differently. The pupils should be led to discover these facts for themselves by the use of such examples.

The distributive law of multiplication over addition enables us to solve a computation in which both the operation of addition and the operation of multiplication are found. The distributive law tells us that we may do such a problem in two different ways - first adding and then multiplying, or first multiplying and then adding. The children themselves should discover the law by doing such problems as the following:

$$5 \times (4 + 3) = 5 \times (4 + 3)$$

$$(5 \times 4) + (5 \times 3) = 5 \times (7)$$

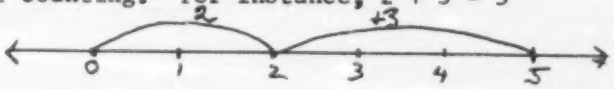
$$20 + 15$$

$$35 = 35$$

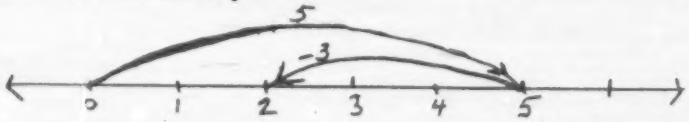
The children should be led to the discovery that in any multiplication involving partial products, the distributive law is used. For instance in multiplying 5 times 24, we first multiply 5 times 4 and then 5 times 20, and add these two partial products to obtain our answer.

III. Operations

Addition and subtraction should be explained in terms of the union of two sets and the difference of two sets. The N notation should be used to indicate the cardinal number of the set, that is, while $\{a,b,c\}$ represents the set having as members the letters "a", "b", and "c", N $\{a,b,c\}$ represents the cardinal number of the set, that is, it is another way of writing the number 3. The "take-away" method using the difference of sets should be used to develop understanding in subtraction. Various visual aids should be used, including the number line. Here we can arrive at the answer in addition and subtraction by a process of counting. For instance, 2 + 3 = 5



For subtraction (the inverse of addition) we count in the opposite direction. For instance, 5 - 3 = 2



The traditional terms "borrowing" and "carrying" have been replaced by the more meaningful term "regrouping," although some textbooks prefer the word "renaming." It is important to begin addition and subtraction problems which involve regrouping by using expanded notation form. For instance we show addition of 27 and 38 in this manner:

2 tens and 7 ones
3 tens and 8 ones
5 tens and 15 ones = 6 tens and 5 ones = 65

Subtraction is done in the same manner. It is vitally important that place value in our number system be stressed at every opportunity.

The basic multiplication facts will be more easily memorized and retained if they are carefully developed with emphasis on understanding. This should be done using the concept of repeated addition of equal addends. Using sets, this would be done by finding the cardinal number of the union of several equivalent sets.

Division should be developed as the inverse operation of multiplication. In division we are looking for a missing factor, when the other factor and the product are given. Division can be introduced by using related sentences such as "2 x ? = 8," and "8 ÷ 2 = ?." The number line should be used also, and a matrix chart of the basic multiplication facts can be used to show division facts and the inverse relationship which division has to multiplication.

Work on fractions is broadened to include non-unit fractions, that is, fractions where the numerator is a whole number other than one. If sets of fractions such as {1/4, 2/4, 3/4} and {1/8, 2/8, 3/8, 4/8, 5/8, 6/8, 7/8} are represented by points on a number line the concept of equivalent fractions will be more clearly understood.

IV. Geometry

Metric geometry is concerned with the size and measurement of geometric figures, whereas non-metric geometry is concerned with no measure or measurement in connection with geometric figures.

All measurement should be done using visual aids, with the pupils participating in handling the materials, such as measuring lengths with a ruler or weighing on a scale. Likewise the concepts of perimeter and area should be made concrete and visual. Pupils should actually measure the perimeter of various geometric figures around them, such as their desk tops, or their books, or the classroom. The concept of area should be developed using squared paper and other visual forms.

The teacher should make the distinction between the representation on paper of a point or line, and the abstract idea of a point and line. A point has no dimension, no matter how small. Likewise a line has only one dimension - length, but the representation of a line on paper must have some width to make it visible. This is analogous to the distinction between number (abstract) and numeral (the symbol representing the number.)

It should be noted that a figure such as a circle or square is a line or set of lines which in turn are made up of sets of points. The region enclosed by these lines is not the figure nor any part of it.

V. Mathematical Sentences

Mathematical sentences (sometimes called number sentences) are statements which describe a mathematical relationship involving numbers. If one or more of the numbers are missing, the mathematical sentence is called an open sentence. The solution set (answer) to an open sentence consists of that number or numbers selected from a prechosen replacement set

which will make the statement true. Open sentences may be either equations or inequalities. They are to be solved intuitively, that is, by trial and error and "educated guessing" without using formal rules. For instance, if $n + 6 = 25$, and the replacement set is the set of all numbers, a pupil may first substitute 12 for n to see if this will make a true statement of the open sentence (or mathematical sentence). This will give $12 + 6 = 25$ which he sees is false, so he tries larger numbers until he arrives at the correct solution 19.

Inequalities are extremely valuable in promoting meaning and understanding of number relationships. The concept of "is less than" and "is greater than" should be used constantly, and illustrated with various types of visual aids such as the number line. For instance, we may have the inequality $n + 5 < 8$ where the replacement set is the set of whole numbers. Here the solution set would be $\{0,1,2\}$.

VI. Estimating and Mental Arithmetic

One of the ways in which a pupil exhibits understanding is to be able to estimate intelligently. Often an unrealistic estimate by a pupil will betray a complete lack of any real understanding of some concept or type of problem. This will alert the teacher to the fact that the explanation and development of the concept has not met with success, and hence reteaching with new approaches and pupil participation are in order.

Mental problems are always valuable for learning number facts, for simple problem solving, and for increasing attention span and improving concentration.

VII. Problem Solving

In order to solve word problems the pupil must be able to translate the relationship expressed in the problem into a mathematical sentence. In order to do this successfully, the pupil must not only know the meaning of the words, but must be able to understand the relationship expressed. Usually it is understanding the mathematical relationship rather than word recognition, that causes difficulty. Where a pupil solves a word problem incorrectly, the trouble may lie in one or more of the four areas: not knowing the meaning of the words; inability to understand the relationship and express it in a mathematical sentence; inability to choose the correct operation; or errors in computation. The teacher must analyze and decide where the difficulty lies, and take necessary steps to help the pupil.

The guided discovery method for teaching problem solving is especially effective. The teacher would ask such questions as "What does the problem ask for?" "What numerical information is supplied to help you find the answer?" "How can we use this information?" "Let us write a mathematical sentence expressing a relationship."

Problem solving is one of the most difficult of topics, and is a real challenge to the creativeness and ingenuity of the teacher.